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ABSTRACT

Raising the level of academic achievement of all students in New Jersey has been a major goal since 1982. As a part of that effort, the focus has changed from the low expectations of the Minimum Basic Skills program to the more rigorous demands of the High School Proficiency Test. This report includes background information on the effort to develop a test of mathematics proficiency, discussions of a framework, the distribution of test items in cognitive domains, and the clusters of skills to be tested. An appendix includes lists of participants in the production of the test and 23 references. (CW)

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Report of the Eleventh-grade High School Proficiency Test Mathematics Skills Development Committee

NEW JERSEY STATE DEPARTMENT OF EDUCATION

SE 051 186

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REPORT OF THE
ELEVENTH-GRADE HIGH SCHOOL PROFICIENCY TEST
MATHEMATICS SKILLS DEVELOPMENT COMMITTEE

Saul Cooperman
Commissioner

Joel Bloom
Assistant Commissioner
Division of General Academic Education

New Jersey State Department of Education
CN 500
Trenton, New Jersey 08625

June 1989

PTM #900.39

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FOREWORD

Raising the level of academic achievement of all students in New Jersey has been a major goal of this administration since 1982. As part of that effort, we have moved from the low expectations of the Minimum Basic Skills (MBS) program to the more rigorous demands of the High School Proficiency Test (HSPT).

Recent history in New Jersey and the nation indicates that the expectations created by a statewide high school graduation test can positively influence the level of achievement of all students, whether from urban, suburban, or rural school districts. In addition, national reports stress the need for workers with increased skill levels in the twenty-first century. For this reason, the New Jersey State Department of Education appointed a panel in 1987 to study the feasibility and efficacy of moving the New Jersey high school graduation test from its current grade nine administration to a grade closer to graduation. At the conclusion of its study, the panel recommended that the administration of the graduation test be moved from the ninth grade to the eleventh grade. This change is part of the response to the increased skill demands of students, including the assessment of higher order skills measuring students' abilities to think critically, solve problems, reason, and make decisions.

During the fall and winter of the 1988-89 school year, reading, mathematics, and writing committees met to identify the skills that high school students will need to function politically, economically, and socially in our increasingly complex technological society. The committees were made up of educators (elementary, secondary, and postsecondary), parents, students, and representatives of our state's business community. Each committee's specific charge was to identify skills to be assessed on the eleventh-grade HSPT. The ninth-grade HSPT was the starting point of the skill identification process. The skills identified by the committees have incorporated and expanded upon the ninth-grade skills to emphasize thinking, problem solving, reasoning, and decision making appropriate for eleventh-graders.

The skills were circulated to all public school districts in the state, as well as to business, parent, student, and educational organizations, for review and comment. Those who responded to the survey indicated support for the committees' work and reinforced the belief that these are the basic skills our graduates will need for employment and continuing education as we move toward the twenty-first century. Responses to the mathematics skills, for example, indicated widespread support for the committee's coherent approach to the testing of students' knowledge of school mathematics. Moreover, there was support for the overriding principles of mathematics assessment detailed in this report. See Appendix A for a listing of reviewers and respondents.

The results of this review process provided direction for clarifying particular skills and clusters. Comments of the reviewers supported the committee's overriding principle that the test should be appropriate for all students who are required to take it for graduation. Moreover, these comments supported the committee's belief that the skills

they identified should emphasize "practical" mathematics. In light of the survey results, the mathematics skills descriptions shown below have been modified as indicated:

PREVIOUS DESCRIPTION	MODIFIED DESCRIPTION
Cluster I-C: represent and use numbers in a variety of equivalent forms (integers, fractions, decimals, percents, exponential notation, and scientific notation) in real-world and mathematical-problem situations	represent and use numbers in a variety of equivalent forms (integers, fractions, decimals, percents, and exponential notation) in real-world and mathematical-problem situations
Cluster I-E: develop and apply number-theoretic concepts, e.g., primes, factors, and multiples, in real-world and mathematical-problem situations	apply such concepts as primes, factors, and multiples in solving problems
Cluster II-C: understand transformations and their effects on geometric figures	understand transformations and their effects on geometric figures and can apply these ideas informally
Cluster IV-D: understand and can apply measures of central tendency	understand and can apply informally statistical concepts, e.g., average, median, mode, range, ranking, and sampling
Cluster IV-E: understand and can use relationships between situations and their representation by means of sample spaces	understand and can use relationships between situations and their mathematical representations
Cluster IV-H: understand and can use algorithms to solve problems	understand and can use algorithms as found in discrete mathematics to solve problems (This skill is focused upon the correctness and efficiency of algorithms rather than upon computational facility.)

The data also provided many sound recommendations concerning the development of item specifications and prototype items.

During the 1989-90 school year, reading, mathematics, and writing committees comprised of school educators will develop sample test items and specifications. These test development committees will have available to

them both the reviewers' comments about the skills and their suggestions about item specifications. In 1990-91, the department will begin three years of "due notice" testing. The first test required for graduation will be administered in December, 1993. Throughout the due notice period, the department will conduct awareness sessions and expanded HSPT institutes to acquaint teachers and administrators with the new skills and strategies for incorporating them in the curriculum.

I commend the members of the committees for their hard work and effort in developing the eleventh-grade HSPT skills. I am grateful to the educators, business people, parents, and students in this state who responded to our call to review the skills.

Saul Cooperman
Commissioner

The Committee wishes to give special recognition to the National Council of Teachers of Mathematics for its Curriculum and Evaluation Standards for School Mathematics which played a central guiding role throughout the deliberations of the committee and during the preparation of this report.

REPORT OF THE
ELEVENTH-GRADE HIGH SCHOOL PROFICIENCY TEST
MATHEMATICS SKILLS DEVELOPMENT COMMITTEE

Background

The Mathematics Skills Committee: Mathematics committee members were chosen to represent a wide range of constituencies. The members of the committee (Appendix B) included:

- representatives of urban, suburban, and rural districts covering all district factor groups;
- district mathematics supervisors and department chairs, school administrators, teachers (including one winner of a Presidential Award for Excellence in Science and Mathematics Teaching), college mathematics educators, members of business and industry, a state PTA officer, a retired educator and a retired businessman, and a high school student.

The Curriculum and Evaluation Standards for School Mathematics (Standards), developed by the National Council of Teachers of Mathematics (NCTM), played a critical role in guiding the planning of the committee. The committee reviewed and used materials related to the National Assessment of Educational Progress (NAEP) testing. In addition, the committee took into account recommendations made by such groups as the Association of Mathematics Teachers of New Jersey, the National Council of Supervisors of Mathematics, the Association of Secondary School Principals, the Mathematical Sciences Education Board, the Board on Mathematical Sciences, the Committee on the Mathematical Sciences in the Year 2000, and the Council of Chief State School Officers.

Moreover, the committee participated in a day-long briefing session on the Evaluation Standards of the NCTM Standards. That briefing session was conducted by Dr. Norman Webb of the University of Wisconsin who was the Chair of the Evaluation Working Group of the NCTM Commission on Standards for School Mathematics.

Throughout its planning, the committee drew upon the services of its consultant on mathematics education, problem solving, and large-scale assessment, Dr. J. Philip Smith, Dean of the School of Arts and Sciences of Southern Connecticut University. Dr. Smith is also a consultant to the Connecticut State Department of Education and the author of CONNCEPT: Study Guide for Mathematics.

Introduction

> Not only in mathematics but also in every other school subject, educators are faced with significant demographic changes and rising expectations for preparing the kind of workforce the country will need in the future. Information-age technology will continue to grow in

> importance. Pressed by rising international competition, American industry will demand improved quality and increased productivity. The world of work in the twenty-first century will be less manual but more mental; less mechanical but more electronic; less routine but more verbal; and less static but more varied.
>
> Communication has created a world economy in which working smarter is more important than just working harder. Jobs that contribute to this economy require workers who are mentally fit--workers who are prepared to absorb new ideas, to adapt to change, to cope with ambiguity, to perceive patterns and to solve unconventional problems. It is these needs, not just the need for calculation (which is now done mostly by machines), that make mathematics a prerequisite to so many jobs. **More than ever before, Americans need to think for a living; more than ever before, they need to think mathematically.** (National Research Council, 1989)

According to the Office of Technology Assessment (1988), businesses no longer seek workers with strong backs, clever hands, and "shopkeeper" arithmetic skills. In fact, A.C. Lewis stresses that the "most significant growth in new jobs between now and the year 2000 will be in fields requiring the most education." (Lewis, 1988)

As the mathematics committee members deliberated, they were guided by the panel recommendations referred to above. Moreover, they were mindful of the recommendations of the ninth-grade HSPT mathematics planning committee that skills selected should emphasize applications of numerical operations, problem-solving experiences that reflect real-life situations, and application of reasoning as a foundation for the mastery of mathematical concepts.

As members of the committee reflected upon the NCTM's 14 Evaluation Standards, they gave particular prominence to the ideas that they thought were most applicable to large-scale assessment of individual student competence. They felt that the following were especially important:

- assessment should determine what students know and how they think about mathematics, rather than only what they do not know;
- assessment should focus on a broad range of mathematical tasks and a holistic view of mathematics, rather than on a large number of specific skills organized by content-behavior;
- assessment should include problem situations involving the application of a number of mathematical ideas, rather than only exercises or word problems requiring one or two skills.

The committee set up the following list of overriding principles to be kept in mind as it selected the cognitive skills to be assessed in the mathematics section of the new eleventh-grade test:

- the test should be appropriate for all the students who are required to take it for graduation;
- whenever possible, process rather than product should be emphasized;
- assessment should emphasize higher-order thinking;
- problem solving should be the focus of school mathematics; therefore, it must be a significant focus of assessment;
- whenever possible, the problems used for assessment should be solvable in a variety of ways;
- assessment should emphasize both the wide variety of careers in which mathematics plays a significant role and also real-world applications, e.g., mathematics in the workplace and consumerism;
- in keeping with the practice of the ninth-grade HSPT, no additional emphasis will be placed on memorizing formulas;
- mathematics is an area of great importance in its own right.

The principles established by the committee reflect a philosophy emphasizing: understanding, not rote learning; applications, not abstractions; problem solving, not drill; thinking, not recall.

Having familiarized themselves with the ninth-grade HSPT mathematics section, the committee members took care to ensure that the clusters of the new test would subsume the content of all the clusters of the ninth-grade HSPT. The five new cluster headings are: Numerical Operations, Measurement and Geometry, Patterns and Functions, Data Analysis, and Fundamentals of Algebra. The following table indicates the relationship between the clusters, old and new.

Ninth-grade HSPT Clusters	**subsumed by**	**Eleventh-grade HSPT Clusters**
COMPUTATION: FRACTIONS, DECIMALS, PERCENTS		NUMERICAL OPERATIONS
NUMBER CONCEPTS		NUMERICAL OPERATIONS
MEASUREMENT AND GEOMETRY		MEASUREMENT AND GEOMETRY
PRE-ALGEBRA		FUNDAMENTALS OF ALGEBRA
PROBLEM SOLVING		ALL CLUSTERS

In particular, just as the isolated, whole-number computation found on the old Minimum Basic Skills test was subsumed entirely in the application problems of the ninth-grade HSPT, the isolated computation skills tested on the ninth-grade HSPT should be embedded entirely in eleventh-grade HSPT items. These items should assess students' ability to apply their knowledge of procedures to solve problems.

A Framework for the Mathematics Section

The committee began the task of designing the eleventh-grade HSPT by developing a matrix. Its organization by content cluster and cognitive domain provided the type of structuring needed for further committee action. Moreover, the matrix depicted below portrays the body of school mathematics to be assessed in terms of interrelated content clusters and cognitive domains. These cognitive domains represent sets of mathematical processes. As such, they provide the theoretical underpinnings of the lists of mathematical skills recommended for assessment on the eleventh-grade HSPT. The lists of skills are in the section entitled "CLUSTERS" beginning on page 9. Guided by a holistic vision of mathematics, illustrated by the interrelatedness of the clusters and domains, the committee sought to develop sets of global objectives rather than traditional behavioral objectives which compartmentalize one's perception of mathematics and, in turn, mathematics instruction and assessment.

CLUSTERS / COGNITIVE DOMAINS	Numerical Operations	Measurement and Geometry	Patterns and Functions	Data Analysis	Fundamentals of Algebra
Conceptual Understanding	●	●	●	●	●
Integration of Mathematical Knowledge	●	●	●	●	●
Procedural Knowledge	●	●	●	●	●
Communication and Reasoning	●	●	●	●	●
Problem Solving	●	●	●	●	●

Distribution of Item Types

The commitee thought it essential to determine the distribution of item types. This was accomplished through a consensus building process in which percentages were assigned to the various content clusters and cognitive domains. This activity and its outcomes helped to refine and crystalize the committee's thinking concerning the role of computation and problem solving in the eleventh-grade HSPT. The committee's recommendations concerning the distribution of item types are displayed in the following tables:

COGNITIVE DOMAINS	Percent Distribution
Conceptual Understanding	20%
Integration of Mathematical Knowledge	25%
Procedural Knowledge	15%
Communication and Reasoning	15%
Problem Solving	25%

CLUSTERS	Percent Distribution
Numerical Operations	30%
Measurement and Geometry	20%
Patterns and Functions	20%
Data Analysis	15%
Fundamentals of Algebra	15%

It should be clear that the committee's pattern of assigning percentages was consistent with its overriding principles, e.g., these percent distributions support the principle that problem solving should be the focus of school mathematics; therefore, it must be a significant focus of assessment.

COGNITIVE DOMAINS

The Evaluations Standards of the NCTM propose changes in the way in which students' proficiency in mathematics is assessed. These recommendations for change, developed in light of current knowledge about evaluation, are intended to revamp many aspects of assessment. As the committee members analyzed the seven Evaluation Standards focusing on assessment of students' understanding of and disposition toward mathematics, they decided to use five categories, the cognitive domains, to subsume these standards.

The committee felt that these five cognitive domains and their associated processes should serve as the foundation of the eleventh-grade HSPT. Their use will help to ensure a comprehensive assessment instrument which will tap more than superficial knowledge. It should be clear that the cognitive domains are not separate or isolated, each independent of the other. Because the domains themselves overlap, they cannot be assessed in isolation.

Conceptual Understanding

Concepts are the essence of mathematical knowledge. Students can make sense of mathematics only if they understand its concepts which have various meanings or interpretations. Understanding mathematical concepts involves more than mere recall of definitions and recognition of common examples; it encompasses the broad range of abilities needed to discriminate between the relevant and the irrelevant attributes of a concept in selecting examples and nonexamples, to represent concepts in various ways, and to recognize their various meanings.

Assessment of students' knowledge and understanding of mathematical concepts should provide evidence that they can:

- **identify, recognize, and understand various meanings and interpretations of concepts;**
- **identify and generate examples and nonexamples;**
- **use models, diagrams, and symbols to represent concepts;**
- **translate from one mode of representation to another;**
- **compare and contrast concepts with other related concepts.**

Integration of Mathematical Knowledge

In mathematics, as in any field, knowledge consists of information plus know-how. Know-how in mathematics requires the ability to use information to

reason and think creatively and to formulate, solve, and reflect critically on problems. A comprehensive assessment of students' mathematical knowledge involves more than simple assessment of how much information they possess about a variety of topics; it includes an assessment of students' ability to use, apply, and communicate that information. The assessment should examine the extent to which students have integrated and made sense of information and whether they can bring it to bear on problems that require reasoning and creative thinking.

The assessment of students' ability to apply and integrate their mathematical knowledge should seek information about their:

- **knowledge and understanding of concepts and procedures;**
- **ability to apply their knowledge to solve problems within mathematics and in other disciplines;**
- **ability to reason and analyze;**
- **ability to understand and use mathematical language to communicate ideas.**

Procedural Knowledge

Although it is important that students know how to execute mathematical procedures in a reliable and efficient manner, knowledge of procedures involves much more than simple execution. Students must know when to apply them, why they work, and how to verify that they give correct answers. They also must understand the concepts underlying a procedure and the logic that justifies it. The assessment of students' procedural knowledge, therefore, should not be limited to evaluating their facility in performing procedures. Assessment should emphasize all of the various aspects of procedural knowledge outlined below.

The assessment of students' knowledge of procedures should provide evidence that they can:

- **recognize when it is appropriate to use a procedure;**
- **give reasons for the steps in a procedure;**
- **execute procedures;**
- **recognize correct and incorrect procedures and results.**

Communication and Reasoning

As is true of any language, to communicate in mathematics means that one is able to use its vocabulary, notation, and structure to express and understand ideas and relationships. Assessment of students' ability to communicate mathematically should be directed both at the meanings they attach to

concepts and procedures of mathematics and at their fluency in describing, understanding, and evaluating ideas expressed through mathematics.

Various types of reasoning are fundamental to doing mathematics but are not always observable in students' work. All students should have explicit opportunities to engage in intuitive, informal reasoning; and, hence, any assessment of students' reasoning abilities should obtain evidence concerning their thinking processes.

The assessment of students' ability to communicate and reason mathematically should provide evidence that they can:

- **express mathematical ideas in writing or by depicting them visually;**
- **understand, interpret, and evaluate mathematical ideas that are presented in written or visual form;**
- **understand and use mathematical language, notation, and structure to represent ideas, describe relationships, and model situations;**
- **use reasoning to recognize patterns and form conjectures;**
- **use reasoning to develop convincing arguments for mathematical statements;**
- **use reasoning to verify conclusions and judge the validity of arguments.**

Problem Solving

If problem solving is to be the focus of school mathematics, it also must be a significant focus of assessment. Problem solving is not a distinct topic, but a process which should permeate all of mathematics instruction. Assessment should determine students' proficiency in all aspects of problem solving. Evidence about students' ability to ask questions, use given information, and make conjectures is essential in evaluating their ability to formulate problems. When students are asked to identify key features of problems, assessment should provide evidence that they can comprehend the question being asked, the nature of the answer being sought, and the mathematical structure of the problem. Also, assessment should provide evidence both on students' use of strategies and problem-solving techniques and on their ability to verify and interpret results.

The assessment of students' ability to solve problems should provide evidence that they can:

- **formulate problems;**
- **identify key features of problems;**
- **apply a variety of strategies to solve problems;**

- **solve problems;**
- **verify and interpret results.**

CLUSTERS

To carry out meaningful assessment of students' mathematical proficiency, it is essential to measure students' skills across the various content clusters. Just as mathematical topics are not isolated but are part of an integrated whole, the clusters must inevitably contain overlapping content. In fact, as the committee recommended, we must adopt a holistic view of mathematics rather than view it as composed of many unrelated topics, a viewpoint that has helped to fragment school mathematics and its assessment into literally thousands of unrelated bits.

Each description of a content cluster provides a brief overview followed by a listing of the skills recommended for assessment of that cluster on the eleventh-grade HSPT. The reader will find that the skills are described intentionally in global terms. This was done to preclude the use of behavioral objectives which have atomized school mathematics and assessment.

I. Cluster: Numerical Operations

This cluster focuses on students' understanding of numbers and their real-world applications. Within this cluster, students are expected to use their conceptual and procedural knowledge in applied situations. The isolated computational skills tested on the ninth-grade HSPT will be embedded in items which assess students' ability to apply their knowledge to solve problems.

Assessment of students' knowledge of numerical operations will provide evidence that the students are able to use conceptual and procedural understanding of numbers, numeration, and operations to:

A. **make estimations and approximations;**

B. **judge reasonableness of results in the context of applications and problem solving;**

C. **represent and use numbers in a variety of equivalent forms (integers, fractions, decimals, percents, and exponential notation) in real-world and mathematical-problem situations;**

D. **apply ratios, proportions, and percents in a wide variety of situations;**

E. **apply such concepts as primes, factors, and multiples in solving problems.**

II. Cluster: Measurement and Geometry

Geometry and measurement interconnect and support each other in many ways. Study of these topics helps students represent and make sense of the world. Geometric models provide a perspective from which students can analyze and solve problems. In this content cluster, the focus is on useful applications of geometric concepts. Students will need to have knowledge of geometric figures and relationships. They should be able to model and visualize geometric figures and use informal reasoning to solve problems.

The study of measurement shows the usefulness and practical application of mathematics. In this cluster, the focus is on the students' ability to describe real-world phenomena using numbers. Students will be asked to identify attributes, select appropriate units, apply measurement concepts, and communicate measurement-related ideas.

Assessment of students' knowledge of measurement and geometry will provide evidence that they:

A. can identify, describe, compare, and classify geometric figures;

B. can visualize and represent geometric figures;

C. understand transformations and their effects on geometric figures and can apply these ideas informally;

D. can represent and solve problems using geometric models;

E. understand and can apply geometric properties and relationships;

F. can classify figures in terms of congruence and similarity and apply these relationships informally;

G. understand measurement, the structure of systems of measurement, and the use of various systems of measurement;

H. can estimate, make, and use measurements to describe and compare phenomena;

I. can select appropriate units and tools to measure to the level of accuracy required in a particular situation;

J. understand the concepts of perimeter, area, surface area, volume, angle measure, capacity, time, temperature, and weight/mass.

III. Cluster: Patterns and Functions

One of the central themes of mathematics is the study of patterns and functions. This study requires students to recognize, describe, and generalize patterns and to build mathematical models that can be used to predict the behavior of real-world phenomena that exhibit the observed pattern. The widespread occurrence of patterns and structure makes the study of patterns

and functions important. **"As a practical matter, mathematics is a science of pattern and order. Its domain is not molecules or cells, but numbers, chance, form, algorithms and change."** (National Research Council, 1989)

Assessment of students' knowledge of patterns and functions will provide evidence that they can:

A. identify, describe, extend, analyze, and create a wide variety of numerical and non-numerical patterns;

B. describe and represent relationships using number sentences, verbal statements, models, tables, graphs, and variable expressions;

C. use patterns and functions to represent and solve problems;

D. model real-world phenomena with a variety of functions represented by, e.g., tables, graphs, and variable expressions.

IV. Cluster: Data Analysis

The committee used this cluster to subsume topics studied in probability, statistics, and discrete mathematics. A focus of this cluster is data representation and analysis across all disciplines. This focus reflects the pervasiveness of these activities in society. Statistical knowledge and the ability to interpret data are essential. In addition, emphasis will be placed on appropriate methods of gathering data, the visual exploration of data, and the evaluation of arguments based on data analysis.

Assessment of students' knowledge of data analysis will provide evidence that they:

A. can systematically collect, organize, and describe data;

B. can construct, read, and interpret tables, charts, and graphs that summarize data from real-world situations;

C. can make inferences and evaluate arguments based on analysis of data;

D. understand and can apply informally statistical concepts, e.g., average, median, mode, range, ranking, and sampling;

E. understand and can use relationships between situations and their mathematical representations;

F. can make predictions based on experimental or mathematical probabilities;

G. can determine the probability of a simple event;

H. understand and can use algorithms as found in discrete mathematics to solve problems. (This skill is focused upon the correctness and efficiency of algorithms rather than upon computational facility.)

V. Cluster: Fundamentals of Algebra

This cluster is not focused on manipulative facility but emphasizes: conceptual understanding, algebra as a means of representation, and algebraic methods of problem solving. Students should master concepts in an informal way as they establish a basic understanding of algebra. Informal mastery should be assessed using, e.g., mathematical models, real data, graphs, and equations.

Assessment of students' knowledge of algebra will provide evidence that they:

A. understand the concepts of variable, expression, and equation;

B. can represent situations and number patterns with tables, graphs, verbal rules, and equations and understand the interrelationships of these representations;

C. can analyze tables and graphs to identify properties and relationships;

D. can solve linear equations and inequalities using concrete, informal, or formal methods;

E. can apply algebraic methods to solve a variety of real-world and mathematical problems.

APPENDIX A

ELEVENTH-GRADE HIGH SCHOOL PROFICIENCY TEST
DISTRIBUTION OF PROPOSED SKILLS
MARCH 1989

Educators

New Jersey State Department of Education

* County Superintendents
* Division Heads
* RCSU Directors

Public School Districts (elementary and secondary)

* Chief School Administrators (285 districts responded)

HSPT Test Development Committees

- Ninth-grade HSPT committee members
- Eleventh-grade HSPT committee members

New Jersey Department of Higher Education

* Chancellor
* Basic Skills Assessment Program

Educational Associations

* New Jersey Association of School Administrators
** New Jersey Business Officials Association
** New Jersey Education Association
* New Jersey Principals and Supervisors Association
* New Jersey School Boards Association
* New Jersey State Federation of Teachers
* Newark Teachers Union

Content-Area Associations

* Art Educators of New Jersey
* Association of Mathematics Teachers of New Jersey
* Educational Media Association of New Jersey
* Foreign Language Educators of New Jersey
* New Jersey Association of Supervision and Curriculum Development
* New Jersey Council for the Social Studies
* New Jersey Council of Teachers of English
** New Jersey Educators of the Gifted and Talented
* New Jersey Language Arts Leaders Association
* New Jersey Library Association
* New Jersey Reading Association

* New Jersey School Counselor Association
* New Jersey Science Supervisors Association
** New Jersey Science Teachers Association
* New Jersey Teachers of English to Speakers of Other Languages/ New Jersey Bilingual Educators
* Vocational Education Association of New Jersey

Business Associations

** Partnership for New Jersey
* Business and Industry Association
** New Jersey Chamber of Commerce

Parent and Student Associations

* New Jersey Congress of Parents and Teachers
* New Jersey Association of Student Councils

* organizations that responded to department survey

** organizations that reviewed materials and chose not to comment

\- copies distributed for information only; no response requested

APPENDIX B

MEMBERS OF THE ELEVENTH GRADE MATHEMATICS SKILLS COMMITTEE

Name	School District/Business
Mr. Andy Baker (Student)	Spotswood High School
Ms. Christine Blake	Camden City Public Schools
Dr. John Butcher	Kean College of New Jersey
Dr. Robert Colangelo	Freehold Township Public Schools
Dr. William DeFabiis	Elmwood Park Public Schools
Mr. George Fricke	New Jersey Bell (Retired)
Ms. Clara Garces	North Bergen High School
Mr. Jerome Harris	Trenton Hign School
Mr. Robert E. Johnson	West Morris Regional High School
Mrs. Roberta Kaar (Recorder)	Spotswood Public Schools
Ms. Ann Marie Kennedy	AT&T
Mrs. Kathleen Kopreski	Vineland High School South
Mr. Richard Kwiatkowski	South River High School
Mr. Paul Lawrence	Franklin Township Public Schools
Mrs. Ruth Mooney	Hamilton Township Public Schools
Dr. Mick Nuspl	East Orange Public Schools
Ms. Marianne Raphaely	NJ Congress of Parents and Teachers
Mrs. Janet Smith	Neptune Public Schools (Retired)
Dr. Thomas Smith	Hunterdon Central Regional School District
Mr. Thomas Tobiasen (Chairperson)	Parsippany-Troy Hills Public Schools
Dr. Robert Urbanski	Middlesex County College
Dr. Philip Smith (Consultant)	Southern Connecticut State University
Dr. Norman Webb (Consultant)	University of Wisconsin-Madison
Dr. Larry Wiley (Consultant)	New Jersey State Department of Education

Mr. Joseph Zahn (Consultant/Liaison to the Committee; prepared the final report)	New Jersey State Department of Education

APPENDIX C

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